Second Order Models and Traffic Data from Mobile Phones

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Abstract

This paper studies the second-order traffic flow models from the aspect of data enabled computations. We consider the *phase transition model* (PTM) (Colombo, 2002) as well as its extension with a Siebel-Mauser type source term (Siebel and Mauser, 2006a), under various assumptions of traffic conditions. We derive the estimation of first-order and second-order traffic quantities based on the vehicle trajectory data. The dataset we utilize is the Next Generation SIMulation (NGSIM, 2006) which contains high quality vehicle trajectories on a segment of I-80, California. The NGSIM data records the location of each passing vehicle at a high precision of every 0.1 second. Our goal is to perform traffic estimation using different second-order fluid models, and analyze how the quality of estimation deteriorates with less frequent sampling of vehicle locations. The findings of this paper will help evaluate the effectiveness of current mobile sensors such as GPS or smart phone – usually with a sampling period of 3 seconds – in reconstructing traffic states.

1 Introduction

1.1 Aw-Rascle-Zhang model and the Siebel-Mauser type source term

The Aw-Rascle-Zhang model was independently developed by Aw and Rascle (2000) and Zhang (2002). The model is established as a two-by-two system of conservation laws. The first equation is based on the conservation of mass:

$$\rho_t + (v\rho)_x = 0 \tag{1.1}$$

for now and sequel, subscripts t and x denotes partial derivative with respect to time and location, respectively. Equation (1.1) is similar to the classical Lighthill-Whitham-Richards model (Lighthill and Whitham, 1955; Richards, 1956), but without assuming that the average velocity v depends only on the average density ρ. The second equation of the model can be derived from a car following model. Taking the reference frame of a driver, and denoting $x(t)$ its position and $v(t)$ its velocity, we write the equation:

$$\frac{d}{dt}[v(t) - u(\rho(t, x(t)))] = \frac{u(\rho(t, x(t))) - v(t)}{T} \tag{1.2}$$

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where the equilibrium velocity u depends only on the average density ρ. $T \in \mathbb{R}_+$ is a relaxation time. In Eulerian coordinates we write:

$$(v - u)_t + v(v - u)_x = \frac{u - v}{T} \tag{1.3}$$

and using $u_t + vu_x = -\rho u' v_x$ we derive:

$$v_t + (v + \rho u'(\rho))_x = \frac{u(\rho) - v}{T} \tag{1.4}$$

which is the second equation.

The second equation is then further manipulated algebraically to reach the form of an homogeneous systems of conservation laws, see Aw and Rascle (2000) for more discussion.

Siebel and Mauser (Siebel and Mauser, 2006a) proposed a variation of the Aw-Rascle-Zhang model by taking into account the reaction time of drivers. The construction is the same as above but gives rise to a second equation as:

$$v_t + (v + \rho u'(\rho))_x = \frac{u(\rho) - v}{T - \tau} \tag{1.5}$$

where τ is a reaction time. Typically τ (in second) varies within $[0.5, 1]$. One should compute T as the time at which the velocity is essentially at equilibrium. In Siebel and Mauser (2006a), T was chosen to be $2/3$ second.

Finally $T - \tau$ is modeled as a factor which can be positive (for very small or very high densities) or negative (for intermediate densities). One has to notice that a negative factor gives rise to stable traffic, while a positive ones produces instabilities.

1.2   Phase transition model

The hyperbolic *phase transition model* (PTM) for traffic flow was introduced by Colombo (2002). It was motivated by the empirical observation that when the vehicle density exceeds certain value, the values of the density-flow pair are scattered in a two-dimensional region, instead of forming certain one-to-one relation as is assumed by the classical LWR model and the fundamental diagram.

The PTM consists of two phases: the free flow phase and the congested phase. In the free flow phase the dynamic is governed by the LWR model

$$\begin{cases} \rho_t + [\rho \cdot v]_x = 0 \\ v = v(\rho) \end{cases} \tag{1.6}$$

The congested phase is governed by the following system.

$$\begin{cases} \rho_t + [\rho \cdot v]_x = 0 \\ q_t + [(q - q^*) \cdot v]_x = 0 \\ v = v(\rho, q) \end{cases} \tag{1.7}$$

where q is identified as the *momentum*. q^* is a given parameter. v is assumed to be a known function of both the density ρ and the momentum q. One choice for such a function is

$$v(\rho, q) = \left(1 - \frac{\rho}{\rho_{jam}}\right) \cdot \frac{q}{\rho} \tag{1.8}$$

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where ρ_{jam} is the jam density. Another choice is

$$v(\rho, q) \;=\; A(\rho_{jam} - \rho) + B(q - q^*)(\rho_{jam} - \rho) \tag{1.9}$$

for some given parameters A and B.

One has a lot of freedom in choosing the second order model in the congested phase. For instance, Goatin (2006) proposed a phase-transition model which employs the Aw-Rascle-Zhang equations in the congested phase. In particular, we can consider the PTM with modified congested velocity by adding a relaxation term as in the Siebel-Mauser model.

1.3 The Next Generation SIMulation (NGSIM)

Initiated by the United States Department of Transportation (US DOT) Federal Highway Administration (FHWA) in the early 2000's, the Next Generation SIMulation (NGSIM, 2006) program collected high-quality primary traffic and trajectory data intended to support the research and testing of the traffic models and open behavioral algorithms.

We take the I-80 data set collected and processed on a segment of Interstate 80 located in Emeryville, California on Apr. 13, 2005. A total of 45 minutes of data are available, segmented into three 15 minute periods. The data set contains vehicle trajectory, instantaneous velocity and acceleration recorded at a high precision of every 0.1 second. By neglecting sensing and processing error, we treat the raw data set as the ground-truth traffic state.

2 Traffic Data Measurements

Using onboard devices such as GPS or smart phone, it is possible to measure the position of a car every δt seconds. Here δt is related to the device's characteristics such as desired precision and transmission capacity. Let us denote $x(t)$ the position and $v(t)$ the velocity at time t. Assume we measure the position at three consecutive shots t_1, t_2 and t_3 with $t_2 - t_1 = t_3 - t_2 = \delta t$ (say around 3 seconds for GPS). From these measurements we can deduce the approximate velocities in the intervals $[t_1, t_2]$ and $[t_2, t_3]$ as:

$$v_{1,2} \;=\; \frac{x(t_2) - x(t_1)}{\delta t}, \qquad v_{2,3} \;=\; \frac{x(t_3) - x(t_2)}{\delta t}. \tag{2.10}$$

The velocity at time t_2 is approximated as

$$v(t_2) \;\sim\; \frac{v_{1,2} + v_{2,3}}{2} \;=\; \frac{x(t_3) - x(t_1)}{2\delta t} \tag{2.11}$$

One also gets

$$\frac{D}{Dt} v(t_2) \;\sim\; \frac{v_{2,3} - v_{1,2}}{\delta t} \;=\; \frac{x(t_3) - 2x(t_2) + x(t_1)}{\delta t^2} \tag{2.12}$$

where $D/Dt = d/dt + v \cdot d/dx$ denotes the material derivative in eulerian coordinates corresponding to the acceleration of the car in Lagrangian ones.

Another important quantity to estimate is the spatial variation of velocity in Eulerian coordinates. For shorter notation, we denote $x_i \doteq x(t_i)$. Assuming a mild variation in time of the eulerian velocity $v(t, x)$, we write:

$$v\left(t_2, \frac{x_2 + x_1}{2}\right) \sim v_{1,2}, \qquad v\left(t_2, \frac{x_3 + x_2}{2}\right) \sim v_{2,3},$$

from which we get, setting $\delta x = \frac{x_3 + x_2}{2} - \frac{x_2 + x_1}{2}$:

$$\frac{\partial}{\partial x} v \left(t_2, \frac{\frac{x_3 + x_2}{2} + \frac{x_2 + x_1}{2}}{2} \right) \sim \frac{v_{2,3} - v_{1,2}}{\delta x}$$

in other words:

$$\frac{\partial}{\partial x} v \left(t_2, \frac{x_3 + 2x_2 + x_1}{4} \right) \sim \frac{v_{2,3} - v_{1,2}}{\frac{x_3 - x_1}{2}} = \frac{2}{\delta t} \frac{x_3 - 2x_2 + x_1}{x_3 - x_1} \tag{2.13}$$

Clearly such approximation is acceptable as long as the variation between $v_{1,2}$ and $v_{2,3}$ is not too large.

3   Traffic Quantities and Model Fitting from Mobile Data

Depending on the choice of the model and the level of approximation we can use the obtained traffic data in many ways to fit an evolutive model (with Kalman filter or other methods). Let us describe in this section some choices which can be used depending on traffic data type, level of accuracy and computational constraints.

3.1   Phase transition model

We depart from the basic phase transition model introduced in Section 1.2. First note that we should determine two critical velocities $V_f > V_c$ and use either LWR model (i.e. the model corresponding to the free flow phase) if $v \geq V_f$; or the second order model if $v < V_c$. In the first case the velocity is sufficient for data fitting. In this paper, we will focus on the second case.

Remark 3.1. *If one wants to use a Newell-Daganzo type fundamental diagram (with constant velocity in the free flow phase), then the equation for v is trivial, but the density can not be reconstructed.*

3.1.1   Phase transition model with source and strongly stable traffic

We fix the phase transition model (Colombo, 2002) with velocity in the congested phase given by

$$v(\rho, q) = A(\rho_{jam} - \rho) + B(q - q^*)(\rho_{jam} - \rho) \tag{3.14}$$

In addition, we add a Siebel-Mauser type source term. Thus the second equation is given by:

$$q_t + (v(q - q^*))_x = \frac{(q - q^*)}{T - \tau}. \tag{3.15}$$

If the traffic is stronly stable, during the data measurement we may assume $\rho_t, \rho_x, q_x \approx 0$ with only q_t non-vanishing. Following Siebel and Mauser (2006a), we assume $T \sim 2/3$ second and $\tau \sim 1$ second. Then $T - \tau \sim -1/3sec$. Notice that by assumption, (3.15) reduces to

$$q_t = \frac{q - q^*}{T - \tau} \tag{3.16}$$

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Then we can derive

$$\frac{D}{Dt}v = \partial_t v\big(\rho(t, x), q(t, x)\big) + v \cdot \partial_x v\big(\rho(t, x), q(t, x)\big) \tag{3.17}$$

$$= v_\rho \cdot \rho_t + v_q \cdot q_t + v \cdot \big(v_\rho \cdot \rho_x + v_q \cdot q_x\big) \tag{3.18}$$

$$= v_q \cdot q_t \tag{3.19}$$

$$= B(\rho_{jam} - \rho) \cdot \frac{q - q^*}{T - \tau} \tag{3.20}$$

$$= -3\,B(\rho_{jam} - \rho)(q - q^*) = -3\,(v - A(\rho_{jam} - \rho)) \tag{3.21}$$

From the strong stability we get:

$$(q - q^*)_t = \frac{q - q^*}{T - \tau} = -3(q - q^*) \tag{3.22}$$

Taking into account only the measurements of v and Dv/Dt, we deduce from (3.21) that

$$(\rho_{jam} - \rho) = \frac{1}{A}\left(v + \frac{1}{3}\frac{Dv}{Dt}\right) \tag{3.23}$$

$$(q - q^*) = -\frac{1}{3}\frac{A}{B}\frac{\frac{Dv}{Dt}}{v + \frac{1}{3}\frac{Dv}{Dt}} \tag{3.24}$$

Using the same notation as in Section 2, the quantity Dv/Dt is computed approximately at time t_2; while to get a value at t_2 of v with good approximation one should set $v(t_2) \sim \frac{v_{1,2}+v_{2,3}}{2}$.

3.1.2 Phase transition model with source and less strongly stable traffic

In this case we are relying on less strong stability of traffic conditions. In other words, we no longer assume that ρ_x vanishes, while still neglecting ρ_t and q_x. Then the equation (3.15) becomes

$$q_t + v_x(q - q^*) = \frac{q - q^*}{T - \tau} \tag{3.25}$$

We can now write

$$\frac{Dv}{Dt} = v_t + vv_x \sim v_q q_t + vv_x = v_q\left(\frac{q - q^*}{T - \tau} - (q - q^*)v_x\right) + vv_x \tag{3.26}$$

By introducing the variables $\hat\rho \doteq \rho_{jam} - \rho$, $\hat q \doteq q - q^*$, we obtain

$$v = A(\rho_{jam} - \rho) + B(q - q^*)(\rho_{jam} - \rho) = \hat\rho\,(A + B\hat q) \tag{3.27}$$

and deduce from (3.27) that

$$\frac{Dv}{Dt} - vv_x = v_q\left(\frac{\hat q}{T - \tau} - \hat q v_x\right) = B\hat\rho\left(\frac{\hat q}{T - \tau} - \hat q v_x\right) \tag{3.28}$$

From (3.27), (3.28) we immediately get the expressions for $\hat\rho$ and $\hat q$ in terms of v, v_x and $\frac{Dv}{Dt}$.

$$\hat\rho = v\frac{v - (T - \tau)\frac{Dv}{Dt}}{A(v - (T - \tau)vv_x)} \tag{3.29}$$

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$$\hat{q} = \frac{A}{B} \frac{(T - \tau)\left(\frac{Dv}{Dt} - vv_x\right)}{v - (T - \tau)\frac{Dv}{Dt}} \tag{3.30}$$

$\frac{Dv}{Dt}$, v_x are given by (2.12) and (2.13) respectively. To approximate v, notice that if one chooses (2.10), the quantity $\frac{Dv}{Dt} - vv_x$ will always vanish, making $\hat{q}$ zero. Thus to avoid such trivial solution, one should approximate v by either

$$v(t_2) \sim v_{1,2} = \frac{x(t_2) - x(t_1)}{\delta t}, \qquad \text{or} \qquad v(t_2) \sim v_{2,3} = \frac{x(t_3) - x(t_2)}{\delta t}$$

3.1.3 Phase transition model without source term

In this case, we consider the PTM without the Siebel-Mauser type source term:

$$\begin{cases} \rho_t + (\rho \cdot v)_x = 0 \\ q_t + ((q - q^*) \cdot v)_x = 0 \\ v(\rho, q) = A(\rho_{jam} - \rho) + B(q - q^*)(\rho_{jam} - \rho) \end{cases} \tag{3.31}$$

We start with the identity

$$\frac{Dv}{Dt} = v_t + v \cdot v_x = v_\rho\, \rho_t + v_q\, q_t + v(v_\rho\, \rho_x + v_q\, q_x).$$

Using (3.31), we deduce

$$\begin{aligned} \frac{Dv}{Dt} &= v_\rho(-\rho_x v - \rho v_x) + v_q(-q_x v - (q - q^*)v_x) + v(v_\rho \rho_x + v_q q_x) \\ &= -v_x(v_\rho \rho + v_q(q - q^*)). \end{aligned}$$

If we consider the measurements of v, Dv/Dt and also v_x as indicated in Section 2 then we can consider the two relations:

$$v = A(\rho_{jam} - \rho) + B(q - q^*)(\rho_{jam} - \rho) \tag{3.32}$$

$$\frac{Dv}{Dt} = v_x(A\rho + B(q - q^*)(2\rho - \rho_{jam})). \tag{3.33}$$

Recall the variables $\hat{\rho} = \rho_{jam} - \rho$ and $\hat{q} = q - q^\star$. Combining (3.32) and (3.33) and solving for ρ we get:

$$\rho = \frac{1}{B\hat{q}}\left(v + \frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}\right) \tag{3.34}$$

So $\hat{q}$ and $v + \frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}$ always have the same sign. Substituting (3.34) into (3.32), we get

$$B^2 \rho_{jam} \hat{q}^2 + B\left(2A\rho_{jam} - \frac{1}{v_x}\frac{Dv}{Dt} - 2v\right)\hat{q} - A\left(v + \frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}\right) = 0 \tag{3.35}$$

The discriminant of the above quadratic equation in the variable $\hat{q}$ is

$$\begin{aligned} \Delta &= 4B^2\left(v + \frac{1}{2}\frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}\right)^2 + 4AB^2\rho_{jam}\left(v + \frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}\right) \\ &= 4B^2\left(\left(v + \frac{1}{2}\frac{1}{v_x}\frac{Dv}{Dt}\right)^2 - A\rho_{jam}v\right), \quad (3.36) \end{aligned}$$

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In order for any meaningful real root of (3.35) to exist, a necessary condition is that

$$\left| v + \frac{1}{2} \frac{1}{v_x} \frac{Dv}{Dt} \right| \geq \sqrt{A\rho_{jam}} \tag{3.37}$$

In the case where the strict inequality of (3.37) holds, two distinct roots $\hat{q}_1$ and $\hat{q}_2$ exist. We distinguish between two cases:

- If $v + \frac{1}{v_x} \frac{Dv}{Dt} - A\rho_{jam} > 0$, then (3.35) has one positive root and one negative root. By (3.34), one should choose the positive root since ρ must be non-negative.

- If $v + \frac{1}{v_x} \frac{Dv}{Dt} - A\rho_{jam} < 0$, then (3.35) has too roots with the same sign. According to (3.34), both roots should be negative. This requires that

$$2A\rho_{jam} - \frac{1}{v_x} \frac{Dv}{Dt} - 2v > 0$$

A sufficient condition for the above to hold is that $\frac{1}{v_x} \frac{Dv}{Dt} > 0$. We will discuss this condition more in the numerical study at Section 4.

3.2 Estimation of mission and fuel consumption

Following Barth et al. (2007), the emission and fuel consumption can be modeled via the *power demand function*. By knowing the vehicle's attributes such as mass, cross-sectional area and rolling resistance coefficient, and the prescribed acceleration and velocity on a particular grade, the total tractive power requirements (in kilowatts) placed on a vehicle is given in simplified form:

$$P^{tractive} = \frac{M}{1000} \cdot v \cdot (a + g \cdot \sin\theta) + \left(M \cdot g \cdot C_r + \frac{\rho}{2} \cdot v^2 \cdot A_c \cdot C_a\right) \cdot \frac{v}{1000} \tag{3.38}$$

$$
\begin{aligned}
M &= \text{vehicle mass}(1200\,kg) \\
v &= \text{vehicle velocity}(m/s) \\
a &= \text{vehicle acceleration}(m/s^2) \\
g &= \text{gravitational constant}(9.81\,m/s^2) \\
\theta &= \text{road grade angle} \\
C_r &= \text{rolling resistance coefficient}(0.005) \\
\rho &= \text{mass density of air}(1.225\,kg/m^3, \text{depending on temperature and altitude}) \\
A_c &= \text{cross-sectional area}(2.6\,m^2) \\
C_a &= \text{aerodynamic drag coefficient}(0.3)
\end{aligned}
$$

The values for certain parameters above are taken from Chan (1997). For simplicity, we assume all vehicles under study have the same attributes, e.g. mass, cross-sectional area, etc. The vehicle emission or fuel consumption can be modeled simply as an affine function of $P^{tractive}$:

$$\text{Emission} = \beta + \alpha\,P^{tractive} \tag{3.39}$$

$$\text{Fuel Consumption} = b + a\,P^{tractive} \tag{3.40}$$

for some constants $\alpha, a > 0, \beta, b$.

4 Numerical Study

The goal of this section is to numerically verify whether data collected via mobile devices such as GPS or smart phone are sufficient to reconstruct first- and second-order traffic quantities. To this end, we will utilize the GNSIM vehicle trajectory data in the following way.

First, we reconstruct the GPS data from the raw data, which records vehicle location every 0.1 second, by taking samples at every N data points. Here N is a positive integer. Since the sampling period of mobile devices is usually 3 seconds, a typical value of N is 30. For comparison purpose, we consider other values of N as well.

Second, we use the reconstructed GPS data from previous step to approximate basic traffic quantities discussed in Section 2. More specifically, let t_i, $i = 1, \ldots, m$ be the time instances at which the location is reported by the GPS, where $t_{i+1} - t_i = \delta t \sim 3$ seconds. Let $x(t_i)$, $i = 1, \ldots, m$ be the corresponding location of the vehicle. The estimates of the quantities of interest v, $\frac{Dv}{Dt}$, v_x read

$$v(t_i) \sim \frac{x(t_{i+1}) - x(t_{i-1})}{2\delta t}, \quad \text{or} \quad v(t_i) \sim \frac{x(t_{i+1}) - x(t_i)}{\delta t} \tag{4.41}$$

$$\frac{Dv}{Dt}(t_i) \sim \frac{x(t_{i+1}) - 2x(t_i) + x(t_{i-1})}{\delta t^2} \tag{4.42}$$

$$\frac{\partial}{\partial x} v \left(t_i, \frac{x(t_{i+1}) + 2x(t_i) + x(t_{i-1})}{4} \right) \sim \frac{2}{\delta t} \cdot \frac{x(t_{i+1}) - 2x(t_i) + x(t_{i-1})}{x(t_{i+1}) - x(t_{i-1})} \tag{4.43}$$

Third, using the estimated values of v, $\frac{Dv}{Dt}$ and v_x in the previous step, we perform the estimation of $\hat{\rho} = \rho_{jam} - \rho$ (first-order) and $\hat{q} = q - q^*$ (second-order) as in Section 3, for the following cases. For shorter notations, we denote by v, $\frac{Dv}{Dt}$ and v_x the right hand sides of (4.41), (4.42) and (4.43), respectively.

- Phase transition model with source and strongly stable traffic:

$$\hat{\rho}(t_i, x(t_i)) = \frac{1}{A} \left(v + \frac{1}{3} \frac{Dv}{Dt} \right) \tag{4.44}$$

$$\hat{q}(t_i, x(t_i)) = -\frac{1}{3} \frac{A}{B} \frac{\frac{Dv}{Dt}}{v + \frac{1}{3} \frac{Dv}{Dt}} \tag{4.45}$$

Here in (4.45), the velocity v is estimated using either expression in (4.41), in order to avoid vanishing denominator $v + \frac{1}{3} \frac{Dv}{Dt}$.

- Phase transition model with source and less strongly stable traffic:

$$\hat{\rho}(t_i, x(t_i)) = v \frac{v + \frac{1}{3} \frac{Dv}{Dt}}{A(v + \frac{1}{3} v v_x)} \tag{4.46}$$

$$\hat{q}(t_i, x(t_i)) = -\frac{1}{3} \frac{A}{B} \frac{\frac{Dv}{Dt} - v v_x}{v + \frac{1}{3} \frac{Dv}{Dt}} \tag{4.47}$$

Notice that if we approximate v with the first expression of (4.41) and approximate $\frac{Dv}{Dt}$, v_x with (4.42), (4.43) respectively, then

$$\frac{Dv}{Dt} - v v_x \sim \frac{x(t_{i+1}) - 2x(t_i) + x(t_{i-1})}{\delta t^2}$$

$$- \frac{x(t_{i+1}) - x(t_{i-1})}{2\delta t} \cdot \frac{2}{\delta t} \cdot \frac{x(t_{i+1}) - 2x(t_i) + x(t_{i-1})}{x(t_{i+1}) - x(t_{i-1})} \equiv 0. \tag{4.48}$$

Thus, to avoid such trivial solution for $\hat{q}$, we will use the second expression of (4.41) for estimating v instead.

- Phase transition model without source term. The physical solutions to the quadratic equation (3.35) can be easily analyzed in the numerical study. Namely, we easily calculate that

$$\frac{1}{v_x}\frac{Dv}{Dt} \sim \frac{\delta t}{2}\cdot\frac{x(t_{i+1})-x(t_{i-1})}{x(t_{i+1})-2x(t_i)+x(t_{i-1})}\cdot\frac{x(t_{i+1})-2x(t_i)+x(t_{i-1})}{\delta t^2} = \frac{x(t_{i+1})-x(t_{i-1})}{2\delta t}$$

which is always positive. In light of the discussion at the end of Section 3.1.3, we proceed as follows. If

$$\left| v + \frac{1}{2}\frac{1}{v_x}\frac{Dv}{Dt} \right| \geq \sqrt{A\rho_{jam}} \tag{4.49}$$

is satisfied, then we compute $\hat{q}$ as

$$\hat{q} = \frac{-B\left(2A\rho_{jam} - \frac{1}{v_x}\frac{Dv}{Dt} - 2v\right) + \sqrt{\Delta}}{2B^2\rho_{jam}}, \tag{4.50}$$

where Δ is given by (3.36). Thus, regardless what is the sign of $v + \frac{1}{v_x}\frac{Dv}{Dt} - A\rho_{jam}$, the solution of ρ is always physical.

Finally, the estimated quantities $\hat{\rho}$, $\hat{q}$ depends on not only the types of traffic flow model and assumptions employed, but also the values of N. In order to compare different estimations on the same ground, we interpolate the continuous-time quantities such as $v(t)$, $\frac{Dv}{Dt}(t)$, $\hat{\rho}(t, x(t))$and $\hat{q}(t, x(t))$ with piecewise affine functions at the discrete points. In order to assess the estimation error, we will compute the relative L^1 error between the quantities obtained with $N > 1$, and the ones obtained with $N = 1$, i.e. the ground truth data.

In the rest of this section, we will present the estimation errors associated with various traffic quantities including velocity and acceleration (Section 4.1); $\hat{\rho}$ and $\hat{q}$ (Section 4.2); and the power demand function which is closely related to vehicle emission and fuel consumption (Section 4.3). We will vary the value of N, or equivalently the sampling period of the vehicle trajectory, to see the impact on the estimation of first-order and second-order traffic quantities.

Our study period is a fifteen minute interval from 4:00 pm to 4:15 pm, on a segment of I-80. A total number of 1724 vehicle trajectories are utilized.

4.1 Reconstructing velocity and acceleration profiles

In the first step of our study, we reconstruct the most basic traffic quantities: velocity and acceleration of the same vehcle, from (4.41) and (4.42). Following the steps explained in the previous subsection, we summarize the mean and standard deviation of the relative error associated with velocity and acceleration below (Table 1). v_{true} refers to the ground-truth data, while $v_{\delta t}$ is the approximated velocity by sampling every δt seconds.

From Table 1 we find that the estimation of velocity, which is a first-order quantity is much more accurate than the estimation of acceleration, which is a second-order quantity. Such a result can be intuitively confirmed by Figure 1, which shows the reconstructed velocity and acceleration trajectories by taking different sampling period. Since the time-trajectory of acceleration has worsened regularity compared to the trajectory of velocity, under-sampling (i.e. $\delta = 3$ seconds) tends to deteriorate the approximation quality of acceleration. But the velocity remains well-approximated in the L^1 sense.

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	Mean (%)				Standard deviation (%)			
Sampling period δt (s)	0.5	1	2	3	0.5	1	2	3
$\dfrac{\left\Vert v_{true} - v_{\delta t} \right\Vert_{L^1}}{\left\Vert v_{true} \right\Vert_{L^1}}$	3.27	4.72	6.36	7.58	1.50	1.81	2.31	2.87
$\dfrac{\left\Vert a_{true} - a_{\delta t} \right\Vert_{L^1}}{\left\Vert a_{true} \right\Vert_{L^1}}$	88.12	98.77	102.23	102.53	9.52	3.10	2.04	2.04

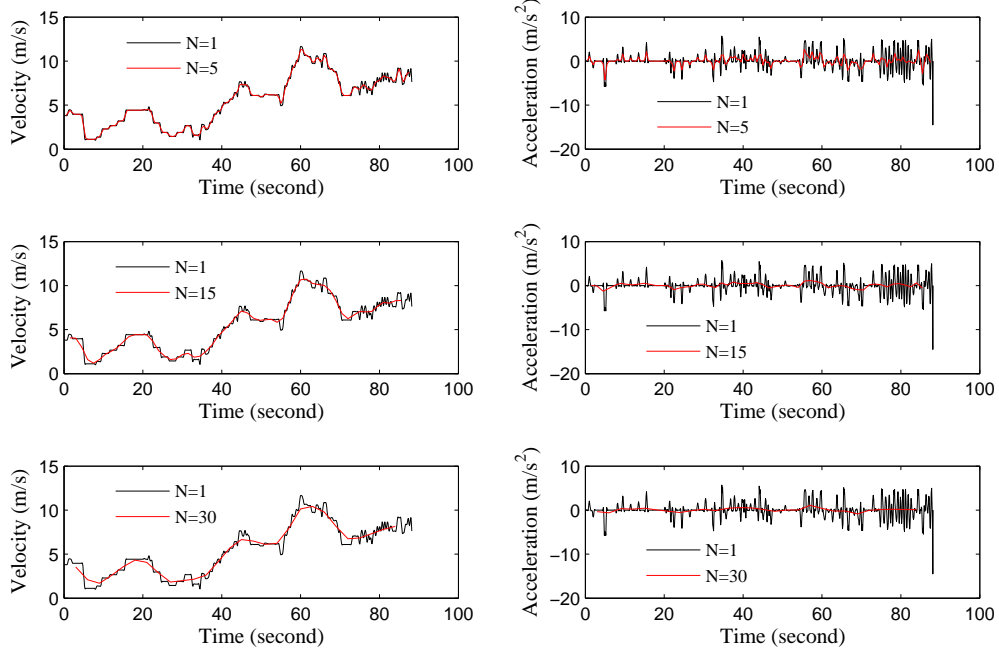Table 1: Relative error using different sampling frequency.

Figure 1: Ground-true and estimated trajectories of velocity (left) and acceleration (right).

4.2 Reconstructing $\hat{\rho}$ and $\hat{q}$

In this subsection, we will estimate $\hat{\rho} = \rho_{jam} - \rho$, which is a first-order quantity, and $\hat{q} = q - q^*$, which is a second-order quantity. ρ_{jam} and q^* are fixed constants. A summary of the utilized dataset shows that the average speed of the 1724 cars under study is 17.5 mile/hour, which indicates that the traffic was in congested phase most of the time. Thus we employ three versions of the phase transition model with the congested phase: PTM with source and strongly stable traffic; PTM with source and less stable traffic; and PTM without source. In the velocity function (3.14), we choose $A = 25$, $B = 1$, $\rho_{jam} = 1$ (vehicle/m). The numerical results reported below are based on these values of parameters. We have repeated the numerical experiment with other values of A, B and ρ_{jam}. The results show no qualitative difference.

4.2.1 PTM with source and strongly stable traffic

The results of relative error are summarized in Table 2, with obvious meaning of notations. Figure 2 and 3 show the reconstructed trajectories associated with a given car.

	Mean (%)				Standard deviation (%)			
Sampling period $\delt t$ (s)	0.5	1	2	3	0.5	1	2	3
$\dfrac{\left\|\hat{\rho}_{true} - \hat{\rho}_{\delta t}\right\|_{L^1}}{\left\|\hat{\rho}_{true}\right\|_{L^1}}$	9.16	10.65	12.11	13.02	6.41	6.38	6.59	6.80
$\dfrac{\left\|\hat{q}_{true} - \hat{q}_{\delta t}\right\|_{L^1}}{\left\|\hat{q}_{true}\right\|_{L^1}}$	95.47	100.36	102.03	102.08	16.76	5.64	2.77	2.56

Table 2: Estimation based on phase transition model with source and strongly stable traffic.
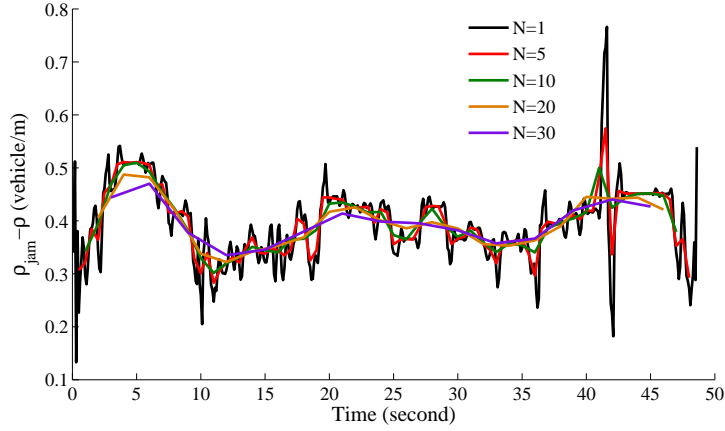

Figure 2: PTM with source and strongly stable traffic: reconstructing the trajectory of $\hat{\rho}$ using different values of N.
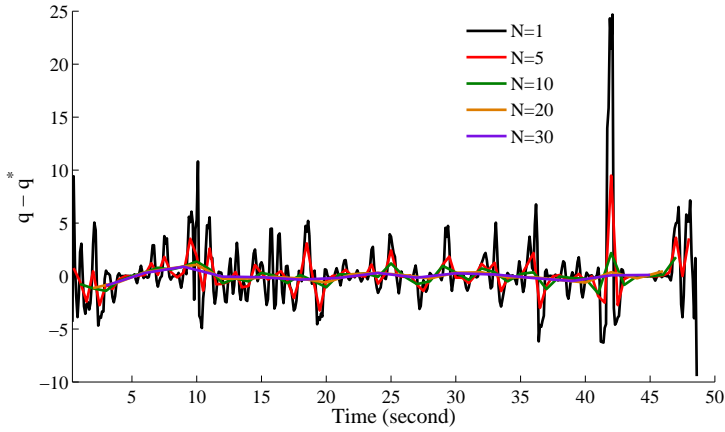

Figure 3: PTM with source and strongly stable traffic: reconstructing the trajectory of $\hat{q}$ using different values of N.

4.2.2 PTM with source and less stable traffic

The results of relative error are summarized in Table 3. Figure 4 and 5 show the reconstructed trajectories associated with a given car.

	Mean (%)				Standard deviation (%)			
Sampling period δt (s)	0.5	1	2	3	0.5	1	2	3
$\dfrac{\left\|\hat{\rho}_{true} - \hat{\rho}_{\delta t}\right\|_{L^1}}{\left\|\hat{\rho}_{true}\right\|_{L^1}}$	12.17	10.17	11.13	12.75	16.36	11.27	10.28	10.55
$\dfrac{\left\|\hat{q}_{true} - \hat{q}_{\delta t}\right\|_{L^1}}{\left\|\hat{q}_{true}\right\|_{L^1}}$	1161	900	772	702	1268	977	1110	1039

Table 3: Estimation based on phase transition model with source and less stable traffic.
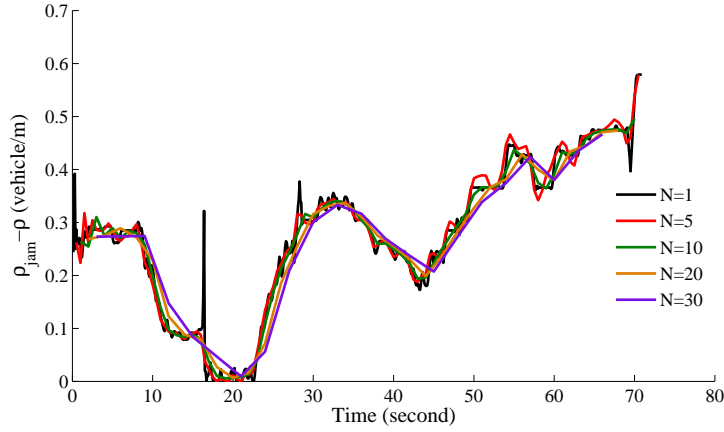

Figure 4: PTM with source and less stable traffic: reconstructing the trajectory of $\hat{\rho}$ using different values of N.
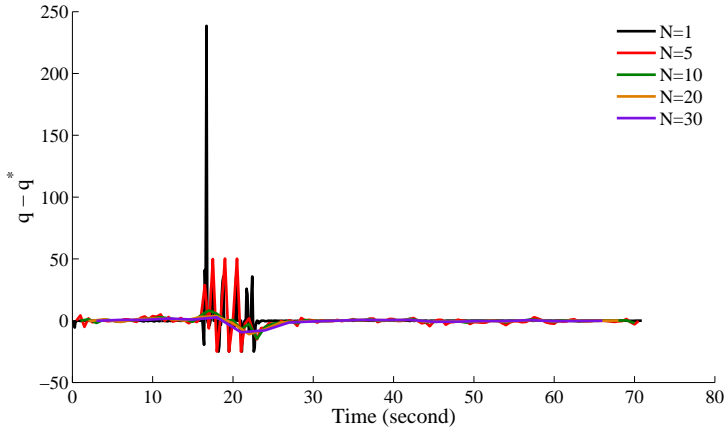

Figure 5: PTM with source and less stable traffic: reconstructing the trajectory of $\hat{q}$ using different values of N.

We notice that for the phase transition model with source and less stable traffic, the error

for second-order quantity $\hat{q}$ is huge. This mainly steps from the fact that the L^1 norm of the ground truth $\hat{q}_{true}$ is very small. Recall (4.47)

$$\hat{q} = -\frac{1}{3}\frac{A}{B}\frac{\frac{Dv}{Dt} - vv_x}{v + \frac{1}{3}\frac{Dv}{Dt}}$$

and calculation (4.48). We see that the above $\hat{q}$ is close (if not identical) to zero, even if slightly different approximations of v, v_x and $\frac{Dv}{Dt}$ are adapted. Such a fact contributes to the small L^1 norm of $\hat{q}_{true}$. An example is shown in Figure 5, where the $\hat{q}$ almost vanishes everywhere.

4.2.3 PTM without source

In this model, the traffic estimates v, v_x and $\frac{Dv}{Dt}$ must satisfy condition (4.49). We selected from the 1724 car trajectories the ones that satisfy such a condition. The number of such trajectories is 177. The results of relative error are summarized in Table 4. Figure 6 and 7 show the reconstructed trajectories associated with a given car.

	Mean (%)				Standard deviation (%)			
Sampling period δt (s)	0.5	1	2	3	0.5	1	2	3
$\dfrac{\left\|\hat{\rho}_{true} - \hat{\rho}_{\delta t}\right\|_{L^1}}{\left\|\hat{\rho}_{true}\right\|_{L^1}}$	0.39	0.56	0.7	0.75	0.22	0.28	0.36	0.42
$\dfrac{\left\|\hat{q}_{true} - \hat{q}_{\delta t}\right\|_{L^1}}{\left\|\hat{q}_{true}\right\|_{L^1}}$	5.41	7.94	9.87	10.49	2.79	3.55	4.37	4.97

Table 4: Estimation based on phase transition model without source.

From Table 4, we see that the estimations of both $\hat{\rho}$ and $\hat{q}$ are quite accurate. This is due to the fact that the velocity variations for these 177 vehicles are relatively small, and the traffic states are relatively stable around these vehicles. This can be partly seen from Figure 6, where the traffic density is quite stable along the vehicle trajectory.
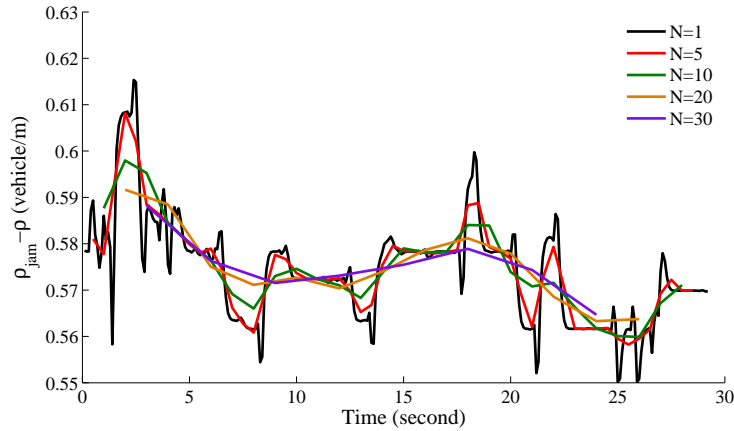

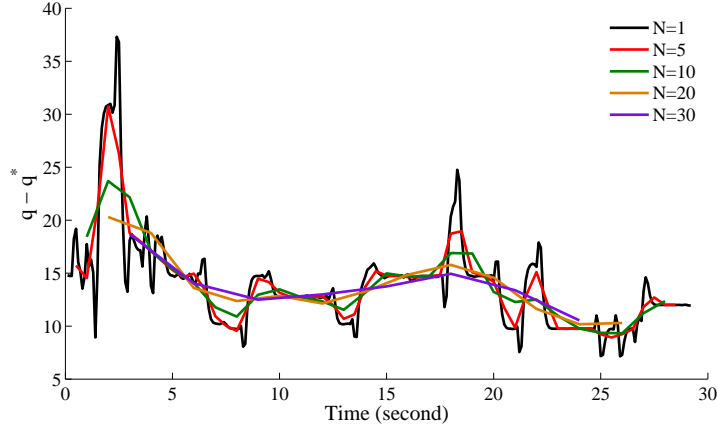Figure 6: PTM without source. Curve fitting of $\hat{\rho}$ using different values of N.

Figure 7: PTM without source. Curve fitting of $\hat{q}$ using different values of N.

4.3 Reconstructing emission and fuel consumption rates

We use the Modal model (3.38) with $\theta \equiv 0$ to calculate the total tractive power requirements. Such a quantity is closely related to vehicle emission and fuel consumption, as argued in Barth et al. (2007). The computation involves car speed v and acceleration $\frac{Dv}{Dt}$.

	Mean (%)				Standard deviation (%)			
Sampling period δt (s)	0.5	1	2	3	0.5	1	2	3
$\dfrac{\left\|P_{true} - P_{\delta t}\right\|_{L^1}}{\left\|P_{true}\right\|_{L^1}}$	87.40	97.60	100.96	101.26	10.09	3.29	1.71	1.63

Table 5: Estimation of power demand.

From Table 5, we conclude that due to the inaccurate approximation of acceleration, the estimation of engine power demand missed the target by almost 100%.

5 Conclusion and Future Research

In this paper, we showed the following.

- Vehicle trajectory data (such as the ones collected using GPS) can be used to estimate Eulerian and Lagrangian traffic quantities such as v, $\frac{Dv}{Dt}$ and v_x. The latter can fit into a second-order fluid model to estimate certain first- and second-order traffic quantities.

- Overall, the first-order quantities such as velocity, density can be accurately estimated even if the sampling period of vehicle location is large (3 seconds). However, the estimation of second-order quantities such as momentum, acceleration, emission and fuel consumption rates deteriorates with long sampling period. In particular, GPS data alone is insufficient to estimate these quantities.

- The phase transition model (PTM) with source and strongly stable traffic seem to be appropriate to treat various traffic scenarios. The PTM with source and less strongly stable traffic is not appropriate to estimate $\hat{q}$. The PTM without source term seems to

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approximate both $\hat{\rho}$ and $\hat{q}$ well, given that the traffic state is stable. In addition, it relies on quite restrictive assumption (3.37) in order to deliver physical solutions.

Some comments and suggestions for future study are as follows.

- Colombo's original two phases approach fits traffic data well, yet the Siebel-Mauser source term allows the model to capture more complicate highways phenomenon (Siebel and Mauser, 2006b).

- The approximation of v_x presented in this paper is more crude than those of v and $\frac{Dv}{Dt}$, and such approximation may lead to large error when the traffic is highly unstable.

- The data fitting depends on used measurements. If one uses only v and Dv/Dt is forced to assume strong local stability of traffic, thus neglecting some dynamics. On the other side the use of v_x permits a better modelling but with rough data.

- The choice of traffic models depends on characteristics of the road segment under consideration (e.g. highways, arterial road and so on), the data quality (e.g. high/low sampling frequency), and the computational constraints. Intuitively, for long and stable highways, the phase transition model with source is a proper choice. For big urban arterial roads, phase transition model without source would be a good model. For short and unstable urban streets, one should use the classical LWR model. The next step is to test these models under various road conditions, as well as data and computational constraints.

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